1) Prove that if $G$ is an undirected bipartite graph with an odd number of vertices, then $G$ is non-Hamiltonian.

2) Show that the Hamiltonian-path problem can be solved in polynomial time on directed acyclic graphs. Give an efficient algorithm for the problem.

3) An independent set of graph $G = (V, E)$ is a subset $V'$ of $V$ such that each edge $e$ is incident on at most one vertex $V'$. The independent set problem is to find the maximum size independent set.

   a) Prove the independent set problem is NP-complete. (Hint: Reduce from the clique problem.)

   b) Suppose that you are given a “black-box” subroutine to solve the decision problem defined in part(a).

   Give an algorithm to find an independent set of maximum size. The running time of your algorithm should be polynomial in $|V|$ and $|E|$, counting queries to the black box as a single step.

4) An Euler circuit for an undirected graph is a path which starts and ends at the same vertex and uses each edge exactly once. A connected, undirected graph $G$ has an Euler circuit if and only if every vertex is of even degree. Give an $O(e)$ algorithm to find an Euler circuit in a graph with $e$ edges provided one exists.

5) Given a $n \times n$ Boolean matrix $N$ can you write an algorithm polynomial in $n$, to rearrange rows and columns such that there is a sub matrix $M$ of size $m \times m$ ($m \leq n$) such that the entries of $M$ are all 1. If you cannot write a polynomial time algorithm prove it.