20.)

We are letting \( a_n \) be the population, in billions of people, \( n \) years after 2010.

a) \( a_n = 1.011a_{n-1}, \) with \( a_0 = 6.9 \)

b) \( a_n = 6.9 \cdot (1.011)^n \)

c) \( a_{20} = 6.9 \cdot (1.011)^{20} \approx 8.6 \) billion people

22.) We let \( a_n \) be the salary, in thousands of dollars, \( n \) years after 2009.

a) \( a_n = 1 + 1.05a_{n-1}, \) with \( a_0 = 50 \)

b) Here \( n = 8 \). We can either iterate the recurrence relation 8 times, or we can use the result of part (c). The answer turns out to be approximately \( a_8 = 83.4 \), i.e., a salary of approximately $83,400.

c) We use the iterative approach.

\[
a_n = 1 + 1.05a_{n-1} \\
= 1 + 1.05(1 + 1.05a_{n-2}) \\
= 1 + 1.05 + (1.05)^2a_{n-2} \\
= (1+1.05+(1.05)^2+\cdots+(1.05)^{n-1}) + (1.05)^na_0 \\
= \frac{((1.05)^n - 1)}{1.05-1} + 50 \cdot (1.05)^n \\
= 70 \cdot (1.05)^n - 20
\]

18) Show that if \( n \) is an integer, then \( n = \lfloor n/2 \rfloor + \lceil n/2 \rceil \).

Proof:

Case 1. \( n \) is even. then \( n/2 \) is an integer, so \( \lfloor n/2 \rfloor + \lceil n/2 \rceil = (n/2)+(n/2) = n \).

Case 2. \( n \) is odd, then \( \lfloor n/2 \rfloor = (n+1)/2 \) and \( \lceil n/2 \rceil = (n - 1)/2 \), so again the sum is \( n \).
22) Prove that \[ \lfloor n/2 \rfloor \lceil n/2 \rceil = \lfloor n^2/4 \rfloor \] for all integers n.

proof:

Case 1. n is even. Then n/2 is an integer, and n^2/4 is also an integer, hence result.

Case 2. n is odd, say n = 2k+1. Then \[ \lfloor n/2 \rfloor = \lfloor k + 1/2 \rfloor = k \]
\[ \lceil n/2 \rceil = \lceil k + 1/2 \rceil = k+1 \]

Therefore the LHS: k(k+1) = k^2 +k;

Now since n = 2k+1; \[ n^2 = (2k + 1)^2 = 4k^2 + 4k + 1, \]
Therefore \[ n^2/4 = k^2 + k +1/4; \]
\[ \lfloor n^2/4 \rfloor = k^2 + k; \text{ LHS = RHS; } \]