Math Methods (COSC 030) HW5 (10 points) 

1) Determine whether \( f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \) is onto if
   a) \( f(m, n) = 2m - n \).
   b) \( f(m, n) = m^2 - n^2 \).
   c) \( f(m, n) = m + n + 1 \).
   d) \( f(m, n) = |m| - |n| \). \( (|\cdot| = 2 \text{ absolute value}) \)
   e) \( f(m, n) = m^2 - 4 \).

14. a) This is clearly onto, since \( f(0, -n) = n \) for every integer \( n \).
   b) This is not onto, since, for example, 2 is not in the range. To see this, if \( m^2 - n^2 = (m - n)(m + n) = 2 \), then \( m \) and \( n \) must have same parity (both even or both odd). In either case, both \( m - n \) and \( m + n \) are then even, so this expression is divisible by 4 and hence cannot equal 2.
   c) This is clearly onto, since \( f(0, n - 1) = n \) for every integer \( n \).
   d) This is onto. To achieve negative values we set \( m = 0 \), and to achieve nonnegative values we set \( n = 0 \).
   e) This is not onto, for the same reason as in part (b). In fact, the range here is clearly a subset of the range in that part.

2) Determine whether each of these functions is a bijection from \( \mathbb{R} \) to \( \mathbb{R} \).
   a) \( f(x) = -3x + 4 \)
   b) \( f(x) = -3x^2 + 7 \)
   c) \( f(x) = \frac{(x + 1)}{(x + 2)} \)
   d) \( f(x) = x^5 + 1 \)
   e) \( f(x) = 2x \)
22. If we can find an inverse, the function is a bijection. Otherwise we must explain why the function is not on-to-one or not onto.

a) This is a bijection since the inverse function is \( f^{-1}(x) = (4 - x)/3 \).

b) This is not one-to-one since \( f(17) = f(-17) \), for instance. It is also not onto, since the range is the interval \((\infty, 7]\). For example, 42548 is not in the range.

c) This function is a bijection, but not from \( \mathbb{R} \) to \( \mathbb{R} \). To see that the domain and range are not \( \mathbb{R} \), note that \( x = -2 \) is not in the domain, and \( x = 1 \) is not in the range. On the other hand, \( f \) is a bijection from \( \mathbb{R} - \{-2\} \) to \( \mathbb{R} - \{1\} \), since its inverse is \( f^{-1}(x) = (1 - 2x)/(x - 1) \).

d) It is clear that this continuous function is increasing throughout its entire domain (\( \mathbb{R} \)) and it takes on both arbitrarily large values and arbitrarily small (large negative) ones. So it is a bijection. Its inverse is clearly \( f^{-1}(x) = \sqrt{x - 1} \).

e) yes \( f(x) = x/2 \)