38) We give a proof by contraposition. If $x$ is rational, then $x = p/q$ for some integers $p$ and $q$ with $q \neq 0$. Then $x^3 = p^3/q^3$, and we have expressed $x^3$ as the quotient of two integers, the second of which is not zero. This by definition means that $x^3$ is rational, and that completes the proof of the contrapositive of the original statement.

46) We give a proof by contradiction. If $\sqrt{2} + \sqrt{3}$ were rational, then so would be its square, which is $5 + 2\sqrt{6}$. Subtracting 5 and dividing by 2 then shows that $\sqrt{6}$ is rational, but this contradicts the theorem we are told to assume.