24. It will be useful to note first that there are exactly 9000 numbers in this range.
   a) Every ninth number is divisible by 9, so the answer is one ninth of 9000 or 1000.
   b) Every other number is even, so the answer is one half of 9000 or 4500.
   c) We can reason from left to right. There are 9 choices for the first (left-most) digit (since it cannot be a 0),
      then 9 choices for the second digit (since it cannot equal the first digit), then, in a similar way, 8 choices for
      the third digit, and 7 choices for the right-most digit. Therefore there are $9 \cdot 9 \cdot 8 \cdot 7 = 4536$ ways to specify
      such a number. In other words, there are 4536 such numbers. Note that this coincidentally turns out to be
      almost exactly half of the numbers in the range.
   d) Every third number is divisible by 3, so one third of 9000 or 3000 numbers in this range are divisible
      by 3. The remaining 6000 are not.
   e) For this and the next three parts we need to note first that one fifth of the numbers in this range, or 1800 of
      them, are divisible by 5, and one seventh of them, or 1286 are divisible by 7. [This last calculation is a little
      more subtle than we let on, since 9000 is not divisible by 7 (the quotient is 1285.71...). But 1001 is divisible
      by 7, and $1001 + 1285 \cdot 7 = 9996$, so there are indeed 1286, and not 1285 such multiples. (By contrast, in
      the range 1002 to 10001, inclusive, which also includes 9000 numbers, there are only 1285 multiples of 7.)] We
      also need to know how many of these numbers are divisible by both 5 and 7, which means divisible by 35.
      The answer, by the similar reasoning, is 257, namely those multiples from 29 \cdot 35 = 1015 to 285 \cdot 35 = 9975.
      (One more note: We could also have come up with these numbers more formally, using the ideas in Section 8.5,
      especially Example 2. We could find the number of multiples less than 10,000 and subtract the number of
      multiples less than 1000.) Now to the problem at hand. The number of numbers divisible by 5 or 7 is
      the number of numbers divisible by 5, plus the number of numbers divisible by 7, minus (because of having
      overcounted) the number of numbers divisible by both. So our answer is $1800 + 1286 - 257 = 2829$.
   f) Since we just found that 2829 of these numbers are divisible by either 5 or 7, it follows that the rest of
      them, $9000 - 2829 = 6171$, are not.
   g) We noted in the solution to part (e) that 1800 numbers are divisible by 5, and 257 of these are also
      divisible by 7. Therefore $1800 - 257 = 1543$ numbers in our range are divisible by 5 but not by 7.

36. There are $2^n$ such functions, since there is a choice of 2 function values for each element of the domain.

16. We can apply the pigeonhole principle by grouping the numbers cleverly into pairs (subsets) that add up to 16,
    namely \{1, 15\}, \{3, 13\}, \{5, 11\}, and \{7, 9\}. If we select five numbers from the set \{1, 3, 5, 7, 9, 11, 13, 15\},
    then at least two of them must fall within the same subset, since there are only four subsets. Two numbers in
    the same subset are the desired pair that add up to 16. We also need to point out that choosing four numbers
    is not enough, since we could choose \{1, 3, 5, 7\}, and no pair of them add up to more than 12.

40. Let $K(x)$ be the number of other people at the party that person $x$ knows. The possible values for $K(x)$ are
    0, 1, \ldots, n - 1, where $n \geq 2$ is the number of people at the party. We cannot apply the pigeonhole principle
    directly, since there are $n$ pigeons and $n$ pigeonholes. However, it is impossible for both 0 and $n - 1$ to be
    in the range of $K$, since if one person knows everybody else, then nobody can know no one else (we assume
    that “knowing” is symmetric). Therefore the range of $K$ has at most $n - 1$ elements, whereas the domain
    has $n$ elements, so $K$ is not one-to-one, precisely what we wanted to prove.