64) We draw the tree, with its root at the top. We show a branch for each of the possibilities 0 and 1, for each bit in order, except that we do not allow three consecutive 0's. Since there are 13 leaves, the answer is 13.

36) Let $K(x)$ be the number of other computers that computer $x$ is connected to. The possible values for $K(x)$ are 1, 2, 3, 4, 5. Since there are 6 computers, the pigeonhole principle guarantees that at least two of the values $K(x)$ are the same, which is what we wanted to prove.

44) Look at the pigeonholes $\{1000, 1001\}, \{1002, 1003\}, \{1004, 1005\}, \ldots, \{1098, 1099\}$. There are clearly 50 sets in this list. By the pigeonhole principle, if we have 51 numbers in the range from 1000 to 1099 inclusive, then at least two of them must come from the same set. These are the desired two consecutive house numbers.

34) Probably the best way to do this is just to break it down into the three cases by sex. There are $C(15, 6)$ ways to choose the committee to be composed only of women, $C(15, 5)C(10, 1)$ ways if there are to be five women and one man, and $C(15, 4)C(10, 2)$ ways if there are to be four women and two men.

Therefore the answer is

$$C(15, 6) + C(15, 5)C(10, 1) + C(15, 4)C(10, 2) = 5005 + 30030 + 61425 = 96,460.$$ 

4) $C(13, 8) = C(13, 5) = 1287$

24) We know that:

$$C(p,k) = \frac{p!}{k!(p-k)!}$$

is a whole number.

Now the numbers $k!$ and $(p-k)!$ are smaller than $p$. Since $p$ is a prime those numbers cannot divide $p$. Therefore $p$ should divide $C(p,k)$. 
28.)

a) To choose 2 people from a set of n men and n women, we can either choose 2 men \( \binom{n}{2} \) ways to do so or 2 women \( \binom{n}{2} \) ways to do so or one of each sex \( n \cdot n \) ways to do so. Therefore the right-hand side counts the number of ways to do this (by the sum rule). The left-hand side counts the same thing, since we are simply choosing 2 people from 2n people.

\[
2 \binom{n}{2} + n^2 = n(n - 1) + n^2 = 2n^2 - n = n(2n - 1) = 2n(2n - 1)/2 = \binom{2n}{2}
\]

b) \( 2 \binom{n}{2} + n^2 = n(n - 1) + n^2 = 2n^2 - n = n(2n - 1) = 2n(2n - 1)/2 = \binom{2n}{2} \)

32. For \( n = 0 \) we want

\[
(x + y)^0 = \sum_{j=0}^{0} \binom{n}{j} x^{n-j} y^j = \binom{0}{0} x^0 y^0,
\]

which is true, since 1 = 1. Assume the inductive hypothesis. Then we have

\[
(x + y)^{n+1} = (x + y) \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j
\]

\[
= \sum_{j=0}^{n} \binom{n}{j} x^{n+1-j} y^j + \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^{j+1}
\]

\[
= \sum_{k=0}^{n} \binom{n}{k} x^{n+1-k} y^k + \sum_{k=1}^{n+1} \binom{n}{k-1} x^{n+1-k} y^k
\]

\[
= \binom{n}{0} x^{n+1} + \sum_{k=1}^{n} \left[ \binom{n}{k} + \binom{n}{k-1} \right] x^{n+1-k} y^k + \binom{n}{n} y^{n+1}
\]

\[
= x^{n+1} + \sum_{k=1}^{n} \binom{n+1}{k} x^{n+1-k} y^k + y^{n+1}
\]

\[
= \sum_{k=0}^{n+1} \binom{n+1}{k} x^{n+1-k} y^k,
\]