8. Find the least integer \( n \) such that \( f(x) \) is \( O(x^n) \) for each of these functions:
   a) \( f(x) = 2x^2 + x^3 \log x \)
   b) \( f(x) = 3x^2 + (\log x)^4 \)
   c) \( f(x) = (x^4 + x^2 + 1)/(x^4 + 1) \)
   d) \( f(x) = (x^3 + 5\log x)/(x^4 + 1) \)

18. Let \( k \) be a positive integer. Show that \( 1^k + 2^k + \ldots + n^k \) is \( O(n^{k+1}) \).

22. Arrange the function \((1.5)^n, n^{100}, (\log n)^3, \sqrt{n} \log n, 10^n, (n!)^2, \) and \( n^{99} + n^{98} \) in a list so that each function is big-\( O \) of the next function.

24. Suppose that you have two different algorithms for solving a problem. To solve a problem of size \( n \), the first algorithm uses exactly \( n^2 2^n \) operations and the second algorithm uses exactly \( n! \) operations. As \( n \) grows, which algorithm uses fewer operations?

26. Give a big-\( O \) estimate for each of these functions. For the function \( g \) in your estimate \( f(x) \) is \( O(g(x)) \), use a simple function \( g \) of smallest order.
   a) \((n^3 + n^2 \log n)(\log n + 1) + (17 \log n + 19)(n^3 + 2)\)
   b) \((2^n + n^2)(n^3 + 3^n)\)
   c) \((n^n + n2^n + 5^n)(n! + 5^n)\)

30. Show that each of these pairs of functions are of the same order:
   a) \( 3x + 7, x \)
   b) \( 2x^2 + x - 7, x^2 \)
   c) \([x + 1/2], x \)
   d) \( \log(x^2 + 1), \log_2 x \)
   e) \( \log_{10} x, \log_2 x \)