8. Find the least integer $n$ such that $f(x)$ is $O(x^n)$ for each of these functions:
   a) $f(x) = 2x^2 + x^3 \log x$
   b) $f(x) = 3x^3 + (\log x)^4$
   c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$
   d) $f(x) = (x^2 + 5\log x)/(x^2 + 1)$

18. Let $k$ be a positive integer. Show that $1^k + 2^k + \ldots + n^k$ is $O(n^{k+1})$.

22. Arrange the function $(1.5)^n, n^{100}, (\log n)^3, \sqrt{n} \log n, 10^n, (n!)^2$, and $n^{99} + n^{98}$ in a list so that each function is big-$O$ of the next function.

24. Suppose that you have two different algorithms for solving a problem. To solve a problem of size $n$, the first algorithm uses exactly $n^2 2^n$ operations and the second algorithm uses exactly $n!$ operations. As $n$ grows, which algorithm uses fewer operations?

26. Give a big-$O$ estimate for each of these functions. For the function $g$ in your estimate $f(x)$ is $O(g(x))$, use a simple function $g$ of smallest order.
   a) $(n^3 + n^2 \log n)(\log n + 1) - (17 \log n + 19)(n^3 + 3n)$
   b) $(2^n + n^2)(n^3 + 3n)$
   c) $(n^n + n^{2n} + 5^n)(n! + 5^n)$

30. Show that each of these pairs of functions are of the same order:
   a) $3x + 7, x$
   b) $2x^2 + x - 7, x^2$
   c) $[x + 1/2], x$
   d) $\log(x^2 + 1), \log_2 x$
   e) $\log_{10} x, \log_2 x$